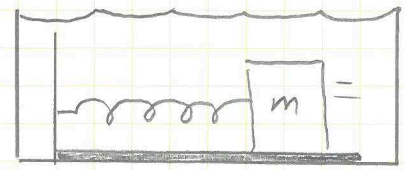
TM5 Pr 3.2 Altered

A 100 g mass on a $10^4 \frac{d}{cm}$ spring oscillates in a resisting medium. After $10\,T_s$, the maximum amplitude is $\frac{1}{2}A_0$.

a) Find β

b) Find ν_s & compare to ν_N



$k = 10^4 \frac{d}{cm}$, $m = 100\,g$

$\omega_N = \sqrt{\frac{k}{m}} = 10\,s^{-1}$

$\nu_N = \frac{\omega_N}{2\pi} = 1.59155\,Hz$

$T_N = \frac{1}{\nu_N} = 0.628357\,s$

$R = \frac{A_0}{A_{10T_s}} = 2$

$n = 10$ (periods)

a) Use the decrement of motion

$$\frac{A_0}{A_{nT_s}} = R = \frac{A_0 e^{-\beta t_0}}{A_0 e^{-\beta(t_0 + nT_s)}} = e^{\beta n T_s}$$

$$\beta n T_s = \ln(R)$$

$$\frac{2\pi n \beta}{\omega_s} = \ln(R)$$

$$\beta = \left[\frac{\ln(R)}{2\pi n}\right]\omega_s = \left[\frac{\ln(R)}{2\pi n}\right]\sqrt{\omega_N^2 - \beta^2}$$

$$\beta^2 = \left[\frac{\ln(R)}{2\pi n}\right]^2 \left(\omega_N^2 - \beta^2\right)$$

$$\left\{1 + \left[\frac{\cancel{2}\ln(R)}{\cancel{4}\pi^2 n^2}\right]\right\}\beta = \left[\frac{\cancel{2}\ln(R)}{\cancel{4}\pi^2 n^2}\right]\omega_N^2$$

$$\beta = \sqrt{\frac{\frac{\ln(R)}{2(n\pi)^2}}{\frac{\ln(R)}{2(n\pi)^2} + 1}}\;\omega_N = \boxed{\sqrt{\frac{\ln(R)}{\ln(R) + 2(n\pi)^2}}\;\omega_N = \beta}$$

For $\omega_N = 10\,s^{-1}$, $R = 2$, and $n = 10$

$$\beta = \sqrt{\frac{\ln(2)}{\ln(2) + 200\pi^2}}\,(10) = \boxed{0.11031\,s^{-1} = \beta}$$

b) Find ω_s

$$\omega_s = \sqrt{\omega_N^2 - \beta^2} = \sqrt{100 - (0.11031)^2} = 9.99939\,s^{-1} = \omega_s$$

$$\nu_s = \frac{\omega_s}{2\pi} = \frac{9.99939}{2\pi} = \boxed{1.59145\,Hz = \nu_s}$$

Or, using the binomial expansion

$$\nu_s = \omega_N\left(1 - \frac{\beta^2}{2\omega_N^2}\right) = \omega_N\left(1 - 6.08\times10^{-5}\right)$$

↳ Very small difference ⇒ Light damping!